Short Baseline QVLBI Demonstrations—Part I

C. C. Chao and S. K. Wong
Tracking and Orbit Determination Section

A. Lubeley
Philco-Ford Corporation

During the period between February 16 and April 11, 1973, three passes of simultaneous two-way and three-way doppler from Pioneers 10 and 11 were obtained from DSSs 11, 12, and 14 at the Goldstone complex. The residuals of the differenced doppler (quasi-very long baseline interferometry–QVLBI) which are free from process noises and effects of transmission media provide a good way to check the stabilities of the tracking system using new rubidium standards (HP 5065) for two-station tracking demonstrations.

Results indicate that the short-term (minute) and medium-term (hour) stabilities agree with expected values (specifications) of about 7 and 3 parts in 10^{13}, respectively. The long-term (month) stability, which is computed from the last two passes of data, exceeds the limited level ($\Delta f/f < 10^{-12}$) required for two-station tracking demonstrations. There are not enough data for a conclusive statement about whether the current system is capable of supporting two-station tracking demonstrations. Further investigation with more data is proposed.

I. Introduction

The new HP5065 rubidium standards which are now in operation at each tracking station have a claimed stability of $\Delta f/f = 7 \times 10^{-13}$ at a 60-s sample rate (Ref. 1). The adequacy of the frequency system using the new standards for QVLBI techniques has not been verified. Simulation studies of MVM'73 and Pioneer 10 indicate that frequency offsets between station frequency systems seriously degrade the potential accuracy of QVLBI doppler data. The desired level of long-term frequency drift is $\Delta f/f = 2 \times 10^{-14}$ (10–30 days) in order to keep the station location errors below the meter level. In Refs. 2 and 3, methods were developed for estimating the frequency offset within required accuracy in the orbit determination (OD) program. Then the tolerance for the long-term offset between frequency systems may be increased to $\Delta f/f = 10^{-12}$. However, these methods assume that the frequency offsets stay constant or vary slowly and linearly with time. To determine whether the systems with the new standard (HP 5065) satisfy these assumptions, a short baseline QVLBI demonstration based on real tracking data from Pioneers 10 and 11 was initiated.

When the simultaneous two-way and three-way data from two stations are differenced, geocentric range rate, together with errors related to the spacecraft process noise (gas leaks, solar pressure), is mostly canceled. Because of the short baseline, the differenced data become very insensitive to transmission media effects, the spacecraft position uncertainty, and errors in the baseline vector. Thus, the residuals of the differenced doppler provide a good way to check the stabilities of the new rubidium standards (HP 5065) in particular, and the entire doppler tracking system in general.

Between February 16 and April 11, 1973, three passes of simultaneous two-way and three-way doppler were obtained from DSSs 11, 12, and 14 at the Goldstone complex. The aim of this demonstration is to analyze these three passes of QVLBI data and to determine the level of stabilities of the frequency system in supporting two-station tracking demonstrations.

II. Background

As shown in Fig. 1, station 1 and station 2 are receiving two-way and three-way doppler, respectively. From Ref. 4, the two observables can be expressed as

$$f_2 = C_3\left[f_q(t_1)\left(1 - \frac{f_R}{f_T}\right) + f_q(t_3) - f(t_1)\right] \quad (1)$$

$$f_3 = C_3\left[f_q(t_1)\left(1 - \frac{f_R^*}{f_T}\right) + f_q^*(t_3) - f(t_1)\right] \quad (2)$$

where

$f_q(t)$ = reference frequency at station 1 at time t

f_R/f_T = received frequency/transmitted frequency at station 1

$C_3 = 96\dfrac{240}{221}$

f_R^*, f_q^* = received frequency and reference frequency at station 2

The QVLBI doppler data are the differenced two-way and three-way doppler as

$$f_3 - f_2 = C_3\left[f_q(t_1)\left(\frac{f_R - f_R^*}{f_T}\right) + f_q^*(t_3) - f_q(t_3)\right] \quad (3)$$

The above equation indicates that QVLBI doppler is sensitive to the bias between two station frequency standards. If we ignore terms higher than $1/C^2$, the above equation can be given in Hamilton and Melbourne's form (Refs. 5 and 6):

$$\begin{aligned} f_3 - f_2 &= K(\dot{\rho}_1 - \dot{\rho}_2) + f_q^*(t_3) - f_q(t_3) \\ &= a + b\sin\omega t + c\cos\omega t + n(t) \end{aligned} \quad (4)$$

where

a = constant part of the error in measuring $f_q^*(t_3) - f_q(t_3)$ over one pass

$b = \dfrac{\omega}{\lambda}\Delta\alpha r_b\cos\delta$

$c = \dfrac{\omega}{\lambda} r_b\cos\delta$

λ = wavelength (λ = 13 cm at S-band)

r_b = baseline projection length

$\Delta\alpha$ = longitude error

$n(t)$ = data noise

A regression is performed on Eq. (4), and the information content of the differenced doppler is expressed in terms of the ability to determine the three coefficients a, b, and c. Quoting Hamilton and Melbourne, Table 1 gives the precision of these estimates for a 4-hour pass of QVLBI doppler with a 1-min sample rate and $\sigma_{\dot{\rho}}$ = 0.4 mm/s (0.003 Hz).

From Table 1, the precision of the parameter c, which contains r_b and cos δ, has been increased by almost 40 times if the constant frequency offset a is known perfectly. This strongly suggests that a careful examination of the stability of the reference frequency system at the two participating stations should be made before doing QVLBI demonstrations.

There are several ways to examine the stabilities between two frequency systems. The most reliable method is to analyze the short baseline QVLBI doppler residuals which contain the information of the overall stabilities of the entire tracking system. After the differencing of simultaneous two-way and three-way doppler from two closely located stations, most of the process noises and transmission media effects should be canceled. The residual of differenced doppler for a single pass can be written as

$$\Delta(f_3 - f_2) = \Delta a + \Delta b\sin\omega t + \Delta c\cos\omega t + n(t) \quad (5)$$

where Δa is the frequency offset, Δb and Δc are the errors due to uncertainties of baseline projection length Δr_b, baseline longitude $\Delta\alpha$, and spacecraft position $\Delta\alpha$, $\Delta\delta$:

$$\begin{aligned} \Delta b &= \left(\frac{\omega}{\lambda} r_b\cos\delta\right)\Delta\alpha + \left(\frac{\omega}{\lambda}\Delta\alpha\cos\delta\right)\Delta r_b \\ &\quad + \left(\frac{\omega}{\lambda} r_b\Delta\alpha\sin\delta\right)\Delta\delta \end{aligned} \quad (6)$$

$$\Delta c = \left(\frac{\omega}{\lambda}\cos\delta\right)\Delta r_b + \left(\frac{\omega}{\lambda}\sin\delta\right)\Delta\delta$$

For reasonable values of $\Delta\alpha$, Δr_b, $\Delta\delta$, we found that

$$\Delta b \approx 0.4 \times 10^{-5}\ \text{Hz}$$

$$\Delta c \approx 0.2 \times 10^{-4}\ \text{Hz}$$

both of which are much smaller than the noise level of the frequency systems.

Thus, the residuals of differenced doppler from two close stations reveal the stabilities of the frequency system.

A criterion for the required long-term stability of the frequency system is established based on simulation studies of long baseline QVLBI doppler. The results of these simulations, which include Pioneer 10 and MVM'73, indicate the desired performance of the station frequency system (standard) for required accuracies of estimated spacecraft position and station location parameters (Fig. 2). To meet the meter level precision in station location, the long-term stability of the frequency standard must be within 2 parts in 10^{14}. If the drift between two frequency standards is constant or slowly varying, it may be estimated in the orbit determination program. Then the tolerance for the frequency stability may be increased to one part in 10^{12} with the help of the closed-loop three-station tracking (Ref. 7). Because a large portion of QVLBI data is at low elevations, refraction effects due to the atmosphere become important. The estimated effect from the uncalibrated atmospheric refraction is also shown in Fig. 2.

III. Data Acquisition and Processing

Three passes of simultaneous two-way (F2) and three-way (F3) doppler data at S-band were obtained from Pioneers 10 and 11 through the stations in the Goldstone complex. The first pass, which was made on February 16, 1973, had 6-h continuous F2 and F3 data at a 1-min sample rate. The two participating stations were DSS 11 and 12. A handover of transmitting station took place in the middle of the pass as requested. The second pass was on March 26, 1973; the two stations were DSS 12 and 14. Only 3 h of data with 1-min count time were taken, and there was no handover of transmitting station. Both the first and second passes of the simultaneous two-way and three-way doppler were obtained from Pioneer 10. The third pass was obtained from Pioneer 11 on April 11, 1973, through the same two stations as the second pass, with about 6 h of 1-min data. A data summary of these three passes is listed in Table 2. The three passes were processed by the current orbit determination program (Ref. 4), and then the two-way and three-way data were differenced by a special program (DIFFER) (Ref. 8). The output of DIFFER, which consists of the residuals (observed − computed) of differenced two-way and three-way doppler data containing the information of frequency stabilities, will be discussed next.

The station standards were measured independently against a reference clock by microwave technique for later comparison.

IV. Results and Discussion

We have shown in Section II that the short baseline QVLBI data are insensitive to both spacecraft position and baseline vector. The completeness of the cancellation of transmission media effects needs to be checked before the results are discussed. For elevation angles higher than 15 deg, the error induced in QVLBI doppler due to an equivalent zenith range error caused by the horizontal gradient and inhomogeneities in the atmosphere (troposphere and ionosphere) can be approximated by the simple relation

$$\Delta\dot{\rho} \cong \frac{\epsilon\rho_z \cos\dot{\gamma}}{\sin^2\gamma}\,\gamma \tag{7}$$

where

$\epsilon\rho_z$ = difference in zenith range corrections between two close stations

$\gamma, \dot{\gamma}$ = elevation angle and angle rate (nearly the same at both stations)

For a 16-km baseline (DSSs 12–14), at the same instant, the difference in zenith range effects due to troposphere and ionosphere should be no greater than a few centimeters. This gives an error less than 0.1 mHz ($\Delta\rho \leq 10^{-4}$ Hz). The estimated error is about the same order of magnitude for space plasma, except when it is very active (Ref. 9). Thus, the residuals of QVLBI data are relatively free from effects of transmission media. According to the results of a short baseline VLBI demonstration (Ref. 10) using the same baseline (DSSs 12–14), 0.1- to 0.2-mHz noises in the data residuals are believed to be due to transmission media. This agrees with the estimated value from Eq. (6).

Data from the first pass were taken on February 16, 1973, at DSS 11 and DSS 12. The two-way and three-way doppler residuals may be due to a series of unmodeled accelerations 3×10^{-9} km/s^2 in magnitude. This type of noise, which was also found in the MM'71 doppler data, is too large to be due to the solar plasma effect and gas leaks (Pioneer 10 is a spinning spacecraft). It might be caused by impacts with small meteorites. Nevertheless, it gives us an additional opportunity to demonstrate the advantage of differenced data types. Figure 3 clearly indicates that the process noise and effects due to transmission have been taken out after the differencing of F2 and F3 data. The statistics of the residuals over this 6-h pass are listed in Table 3. The residuals of F3 − F2 have mean values of −0.0436 Hz in the first 3 h and 0.0423 Hz in the second 3 h of the pass after the handover of transmitting stations. From Eq. (3), this change of sign strongly indicates that a frequency offset existed between the two-station frequency system. The standard deviation of the residuals of F3 − F2 from the mean is 0.0027 Hz, which is consistent with the stability of the new rubidium standards, $\Delta f/f \simeq 10^{-12}$ (Ref. 1).

The second pass, on March 26, was acquired from a different pair of tracking stations (DSSs 12 and 14). As indicated in Fig. 4, this pass of data exhibited low process noise. The data noise of the differenced doppler did not show much improvement over that of the F2 and F3 data (Table 4). A bias of −0.0079 Hz exists in the F3 − F2 data, with a smaller deviation, $\sigma = 0.0018$ Hz.

The third pass was obtained from Pioneer 11 on April 11 at DSSs 12 and 14. During this pass, the spacecraft was executing a midcourse correction of 38 m/s in velocity change, consisting of one long continuous burn and several small jet pulses. The two-way and three-way doppler data were received 10 to 20 s out of synchronization during half of the 6-h pass. The estimated error in QVLBI doppler data induced by this and the 38-m/s change in velocity is around 0.015 Hz. The midcourse correction and the out-of-synchronization receiving time provide an explanation for the much noisier F2, F3, and F3 − F2 residuals compared to those of the preceding two passes.

Table 5 shows a bias of −0.0189 in F3 − F2 residuals with a large deviation ($\sigma = 0.0079$ Hz). The noise can be seen in the scattering of two-way, three-way, and differenced doppler data presented in Fig. 5. No handover of transmitting stations took place.

To check the medium-term drift in frequency, a straight line was fitted to the F3 − F2 residuals of each of the three passes. The values of the constant offsets and slopes are shown in Table 6. Because of the short data length (3 to 6 h), the time $t = 0$ is picked up at the midpoint of each pass to minimize the correlation between the constant and the slope of a straight line. The constant terms or the means of the offset can be determined with uncertainties from 0.13 to 0.5 mHz. The slope of the first pass is the largest, with 0.47 ±0.28 mHz/h. For the last two passes, the values are smaller than their uncertainties (see Table 6). The long-term drift can be seen from the means of the last two passes, which were obtained from the same two stations. A change of 11.2 ±0.63 mHz in 16 days is found; this corresponds to a slope of 0.7 ±0.04 mHz/day.[1]

[1]There was no reset of frequency standards during this interval.

When we compared the short-term (minutes) stabilities obtained from this demonstration with the specifications of frequency standards, we found that the expected value of HP 5065 rubidium is about 1.4 times smaller than our results from differenced doppler data (Fig. 6). The discrepancy can be explained by the fact that the QVLBI residuals contain the noise from two rubidium standards. The medium-term (hours) stabilities estimated from the values of the slopes of fitted straight lines agree with the results of a similar short baseline VLBI demonstration (Ref. 10). The results of QVLBI data provided some new information on the long-term (days) performance of one frequency system relative to another. Figure 6 shows the long-term relative stability to be around 5 parts in 10^{12}, which apparently exceeds the desired level for orbit determination as shown in Fig. 2.

During the first two passes, microwave measurements were made (Ref. 11). The values of the relative frequency offsets between two station standards are:

(1) First pass, 3-h interval: $f_{11} - f_{12} = -0.036 \pm 0.02$ Hz.

(2) Second pass, 10-h interval: $f_{14} - f_{12} = -0.015 \pm 0.01$ Hz.

The relative frequency offsets measured by microwave agree with the values determined from differenced doppler data even though the microwave measurements have a large uncertainty.

V. Concluding Remarks and Proposed Studies

The results from differencing three passes of simultaneous two-way and three-way doppler data show that the short-term (1-min averaging time) relative stability between two tracking systems using HP 5065 frequency standards is around 1 part in 10^{12}. The medium-term (hours) and long-term (days) stabilities are about 3 parts in 10^{13} and 5 parts in 10^{12}, respectively. The long-term stability, which is based on only two passes of data, exceeds the desired level for two-station tracking demonstrations. In order to enhance our understanding of the long-term performance of the frequency systems and their stabilities, excluding the frequency standards (HP 5065), more passes of short baseline QVLBI data are needed. The required tracking patterns for continued short baseline QVLBI doppler demonstrations are proposed in Table 7.

Acknowledgments

The authors wish to thank Jay Curtright for conducting the microwave measurement and for his helpful discussions. Thanks are also given to David Farless and Robert Ryan for scheduling the two-station tracking. Special appreciation is due to D. W. Trask for his thorough review of this article and useful suggestions.

References

1. *Hewlett-Packard Rubidium Frequency Standard 5065A*, Technical Data, Apr. 15, 1971, p. 283.

2. Ondrasik, V. J., and Rourke, K. H., "Applications of Quasi-VLBI Tracking Data Types to the Zero Declination and Process Noise Problems," in *The Deep Space Network Progress Report*, Technical Report 32-1526, Vol. IV, Jet Propulsion Laboratory, Pasadena, Calif., Aug. 15, 1971.

3. Mulhall, B. D., et al., *Report on the Two-Station Doppler (QVLBI) Demonstrations Conducted with Mariner IX, Part 1, Batch Filtering*, TM 391-413, 1973 (JPL internal document).

4. Moyer, T. D., *Mathematical Formulation of the Double Precision Orbit Determination Program*, Technical Report 32-1527, Jet Propulsion Laboratory, Pasadena, Calif., May 15, 1971.

5. Hamilton, T. W., and Melbourne, W. G., "Information Content of a Single Pass of Range Rate Observables," in *The Deep Space Network*, Space Programs Summary 37-39, Vol. III, Jet Propulsion Laboratory, Pasadena, Calif., May 31, 1966, p. 18.

6. Chao, C. C., *A Preliminary Comparison of Two-Station Tracking Data Types*, TM 391-262, Dec. 8, 1971 (JPL internal document).

7. Chao, C. C., *A Simple Analysis of QVLBI Data Obtained from Three Widely Separated Stations*, TM 391-440, Apr. 1973 (JPL internal document).

8. Johnson, D. E., *User's Guide to Differenced Partial Program*, TM 391-333, June 9, 1972 (JPL internal document).

9. von Roos, O. H., private communication.

10. Thomas, J. B., et al., "Radio Interferometry Measurements of a 16-km Baseline with 4-cm Precision," paper presented at the American Geophysical Union Annual Meeting, 1972.

11. Curtright, Jay, private communication.

Table 1. Precision of coefficient estimates, σ_p = 0.4 mm/s, 1-min samples

Estimated coefficients	σ_a/ω, m (σ_a, mHz)	σ_b/ω, m	σ_c/ω, m
a, b, c	6.8 (3.4)	1.2	7.6
b, c	–	1.2	0.2

Table 2. Tracking data summary

Pass	Date	Participating DSSs	Simultaneous doppler data distribution (1-min sample rate), h		Spacecraft
			Two-way	Three-way	
1	Feb. 16, 1973	11	3	3	Pioneer 10
		12	3	3	
2	Mar. 26, 1973	12	3		Pioneer 10
		14		3	
3	Apr. 11, 1973	12	6		Pioneer 11
		14		6	

Table 3. Statistics of residuals (February 16, 1973)

	Mean, Hz		σ, Hz		rms		Number of data points	
	DSS 11	DSS 12	DSS 11	DSS 12	DSS 11	DSS 12	DSS 11	DSS 12
F2	−0.0828	−0.0773	0.00605	0.0070	0.0829	0.0776	219	121
F3	−0.1210	−0.0405	0.0071	0.0059	0.1212	0.0409	121	221
F3 − F2	−0.0436	−0.0423	0.0027	0.0027	0.0437	0.0424	119	217

Table 4. Statistics of residuals (March 26, 1973)

	Mean, Hz	σ, Hz	rms	Number of data points
F2 DSS 12	0.01079	0.00214	0.01099	231
F3 DSS 14	0.00273	0.00226	0.00355	187
F3 − F2	−0.00786	0.00184	0.00807	184

Table 5. Statistics of residuals (April 11, 1973)

	Mean, Hz	σ, Hz	rms	Number of data points
F2 DSS 12	1.003	0.029	1.0035	319
F3 DSS 14	0.985	0.030	0.9855	333
F3 − F2	−0.0189	0.0079	0.0204	313

Table 6. A straight line[a] fitted to F3 − F2 residuals

Pass	Date	a, mHz	b, mHz/h
1	February 16, 1973	−42.26 ±0.18	0.467 ±0.276
2	March 26, 1973	−7.85 ±0.13	−0.097 ±0.125
3	April 11, 1973	−19.03 ±0.50	−0.055 ±0.320

[a]Δ (F3 − F2) = $a + bt$

Table 7. Accomplished and proposed short baseline QVLBI demonstrations

	Accomplished demonstration	Proposed demonstration
Data length	Three passes in 52 days, with 3–6 h data in each pass. (Only two passes can provide long-term stability.)	Ten passes in 30 days (evenly distributed), with 6 to 10 h of data in each pass. (This will give a clear picture of the long-term performance.)
Participating stations	DSSs 11, 12, and 14.	DSSs 12 and 14 (entire frequency system). DSSs 42 and 43. (Because they use a common frequency standard, the differenced data will show the stabilities of other parts of the system.)
Frequency standards	HP 5065A.	HP 5065A. (No reset should take place during the period of tracking.)
Simultaneous two-way and three-way	10 and 20 s off synchronization during the last pass.	Maintain synchronization of time tag throughout the tracking period.
Independent measurement	Microwave measurements were made for the first two passes.	Continuous microwave measurement.
Spacecraft	Pioneers 10 and 11, with one pass during midcourse maneuver.	Pioneer 10 only, with no midcourse maneuver during the tracking period.

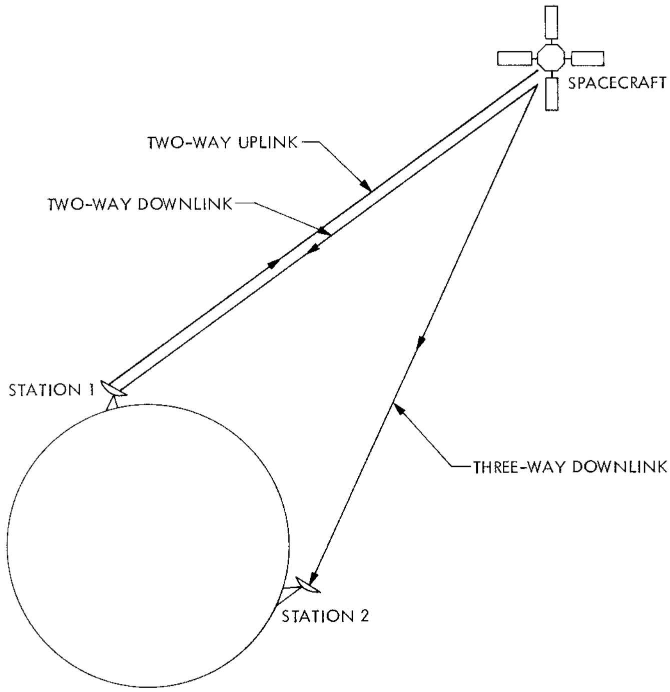

Fig. 1. Geometry of two-way and three-way doppler tracking

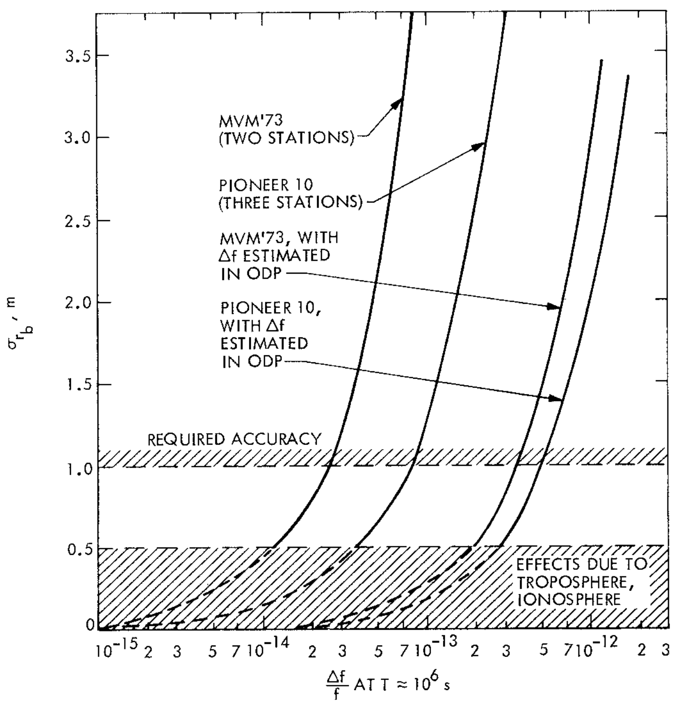

Fig. 2. Estimated effect of frequency offset on relative station location r_b uncertainties

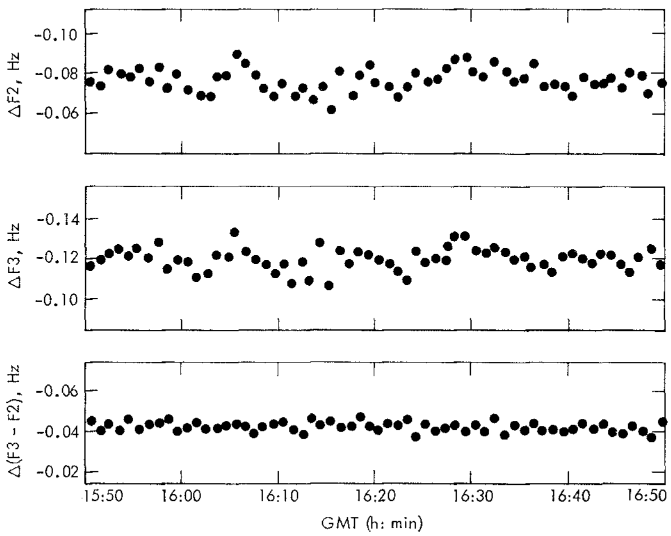

Fig. 3. Residuals of two-way, three-way, and differenced doppler on February 16, 1973 (DSSs 11, 12)

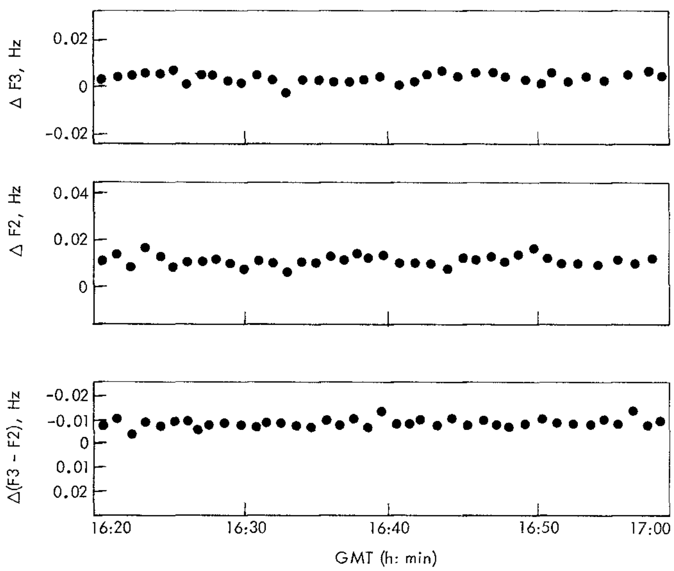

Fig. 4. Residuals of two-way, three-way, and differenced doppler on March 26, 1973 (DSSs 12, 14)

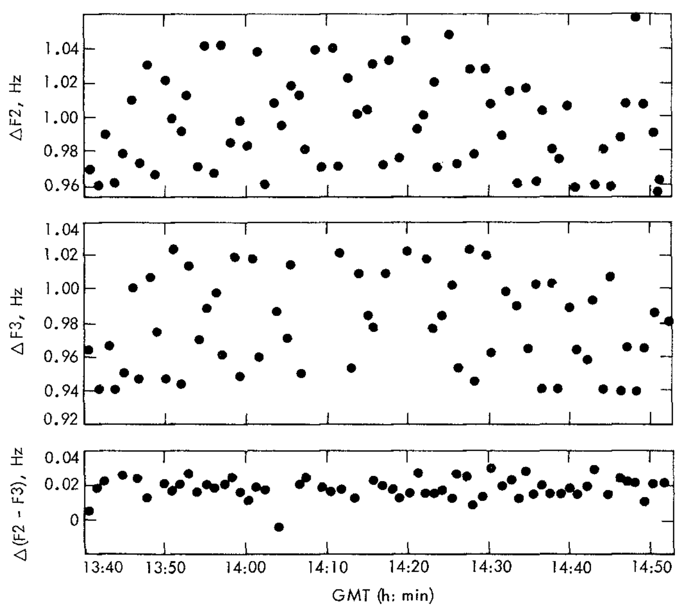

Fig. 5. Residuals of two-way, three-way, and differenced doppler on April 11, 1973 (DSSs 12, 14)

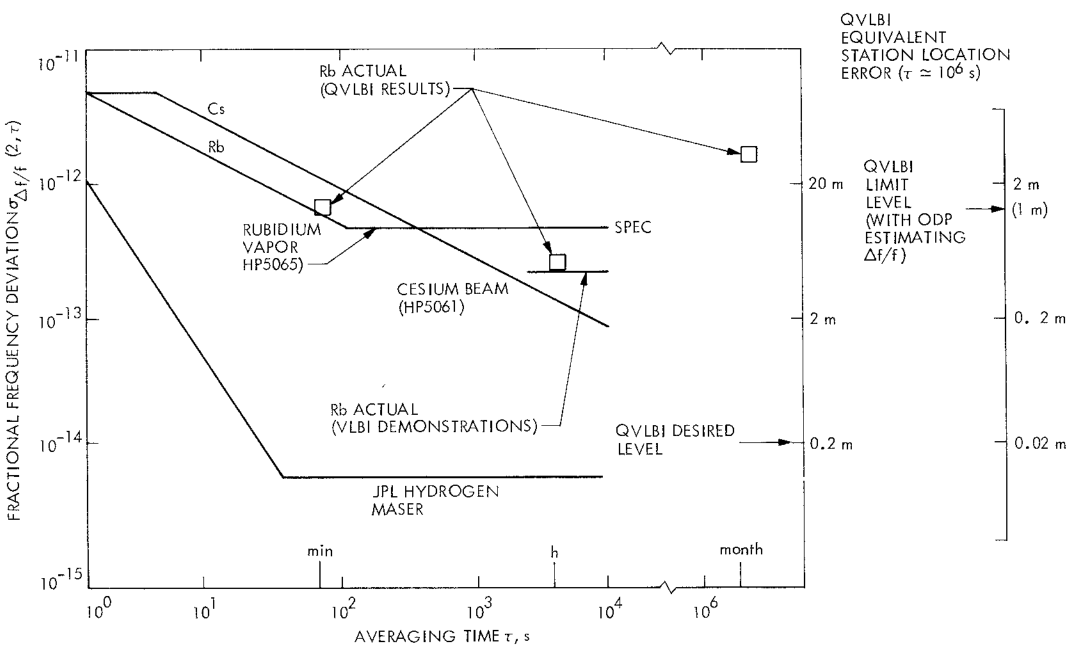

Fig. 6. Short baseline QVLBI demonstration—frequency system